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10 bins

20 balls

4 max balls per bin

A. How many ways can the balls be distributed?

$$= C\binom{10+20-1}{10-1} - C\binom{10}{1}\cdot C\binom{10+20-1-5}{10-1} + C\binom{10}{2}\cdot C\binom{10+20-1-10}{10-1} - C\binom{10}{3}\cdot C\binom{10+20-1-15}{10-1}$$

$$+ C\binom{10}{4}\cdot C\binom{10+20-1-20}{10-1}$$

$= 856945$

B. What is the probability exactly 0 bins will be empty?

↳ same as asking probability all full

↳ same as dropping 1 ball into each bin then determining the # of ways the remaining 10 balls can be distributed amongst the 10 bins

10 bins

10 balls

3 max balls per bin

$$= C\binom{10+10-1}{10-1} - C\binom{10}{1} C\binom{10+10-1-4}{10-1} + C\binom{10}{2} C\binom{10+10-1-8}{10-1}$$

$= 44,803$ ways all bins can be filled

$\frac{44,803}{856,945} = .052282 = P$(all bins contain at least one ball)

c. What is prob exactly 1 bin will be empty?

↳ Same as removing one bin from consideration, dropping one ball into each of nine bins, the determining the number of ways the remaining 11 balls can be distrib amongst the nine bins

9 bins

11 balls

3 max balls per bin

$$= \left[C\binom{9+11-1}{9-1} - C\binom{9}{1}\cdot C\binom{9+11-1-4}{9-1} + C\binom{9}{2}\cdot C\binom{9+11-1-8}{9-1}\right]\cdot C\binom{10}{10-9}$$

= 236,070 ways the nine bins can be filled after dropping one ball into each

$$\frac{236{,}070}{856{,}945} = .275479 = P(\text{exactly one bin is empty})$$

Generic formula for total arrangements

n bins

m balls

r-1 bin maximum

$$\#\text{ of arrangements} = C\binom{m+n-1}{n-1} - C\binom{n}{1}\cdot C\binom{m+n-1-r}{n-1} + C\binom{n}{2}\cdot C\binom{m+n-1-2r}{n-1} - \ldots\ldots$$

continue until $(m+n-1-r)$ exceeds $(n-1)$